Section 4.5
Linearization and Newton's Method

Linear Approximation - using the tangent to approximate the function

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a)$$

defines the linearization of f at a.

The approximation $f(x)$ is approximately $L(x)$

The point $x = a$ is the CENTER of the approximation

Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$, and use it to approximate $\sqrt{1.02}$ without a calculator.

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{1+x}}$

$f'(0) = \frac{1}{2\sqrt{1}} = \frac{1}{2} = m$

$y - 1 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + 1$

$f(0) = \sqrt{1+0} = 1$

$(0, 1)$

≈ 1.01

Use linearization to approximate at x = 2 for

$$f(x) = x^2 - \frac{1}{x}$$

for $x = 2.2$

$f'(x) = 2x + \frac{1}{x^2}$

$f(2) = 4 - \frac{1}{2} = 3.5$

$(2, 3.5)$

$f'(2) = 4 + \frac{1}{4} = 4.25 = m$

$y - 3.5 = 4.25(x - 2)$

$y = \underline{4.25x - 5}$

≈ 4.35

Find the linearization of f(x) = cos x at x = π/2 and use it to approximate cos 1.75 without a calculator.

$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\left(\frac{\pi}{2}, 0\right)$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1 = m$

$y = -1\left(x - \frac{\pi}{2}\right)$

$y = -x + \frac{\pi}{2} = -1.75 + \frac{\pi}{2} = -.17$

Differentials

Let $y = f(x)$ be a differentiable function.
The differential dx is an independent variable.

The differential dy is

$$dy = f'(x)\, dx$$

Find dy if $f(x) = 3x^2 - 2x$ and $x = 4$ and $dx = .2$

Find the differential dy and evaluate dy for the given values of x and dx.

a.) $y = x^5 + 37x, \quad x=1, \quad dx = 0.01$

Find the differential dy and evaluate dy for the given values of x and dx.

b.) $y = \sin 3x$, $x = \pi$, $dx = -.02$

Find the differential dy and evaluate dy for the given values of x and dx.

c.) $x + y = xy$, $x = 2$, $dx = 0.05$

Find the change in volume of a sphere when the radius is increased from 3 to 3.1 inches

The radius r of a circle increases from a = 10 m to 10.1 m. Use dA to estimate the increase in the circle's area A.

Inflating a bicycle tire changes its radius from 12 inches to 13 inches. Use differentials to estimate the change in the perimeter of the tire.

In the late 1830s, the French phsyiologist Jean Poiseuille discovered the formula we use today to predict how much the radius of a partially clogged artery has to be expanded to restore normal flow. His formula was

$$V = k\,r^4$$

says that the volume V of fluid flowing through a small pipe or tube in a unit of time at a fixed pressure is a constant times the fourth power of the tube's radius r. How will a 10% increase in r affect the V?